# Reliability Model for Test Preparation Study

Here we describe the planned approach for assessing the impact of test preparation on test score precision. This approach estimates the test prep precision impact from the covariance matrix of observed subtest scores. This covariance matrix among 20 subtests is constructed from two administrations of the ASVAB to a group of military applicants: 10 subtests of the first battery administered before test preparation and 10 subtests of an alternate form administered after test preparation. Test score data will be obtained from examinees who choose to take the test twice and happen (as a matter of their own choosing) to engage in test preparation inbetween the two administrations.

Let $\Phi$ represent the $10 \times 10$ correlation matrix of subtest (GS, AR, ..., AO) scores that occur before (and therefore are unaffected by) test preparation. Then the elements of $\Phi$ (denoted by $\phi_{ij}$, for $i = 1, ..., 10; j = 1, ..., 10$) represent the correlations among the subtests under pre test-preparation conditions. Further, we construct a $20 \times 20$ diagonal matrix

$$R = \begin{bmatrix} 1 & & & & & \\ & \ddots & & & \mathbf{0} & \\ & & 1 & & & \\ & & & \rho^*_{\mathrm{GS}} & & \\ & \mathbf{0} & & & \ddots & \\ & & & & & \rho^*_{\mathrm{AO}} \end{bmatrix},$$

where $\rho^*_{\mathrm{GS}}, ..., \rho^*_{\mathrm{AO}}$ denote the reliability indices associated with the 10 subtests of the alternate form administered *after* test preparation. Then using the classical test theory formula for modeling the effects of measurement error[1] (i.e., the correction for attenuation formula), the observed covariance matrix $\Sigma$ among all 20 subtests can be expressed as

$$\Sigma = \Lambda R^{1/2} J \Phi J' R^{1/2} \Lambda \,, \qquad (1)$$

[1]In the present case, we equate the more general concept of *measurement error* to the special case of random errors introducted by test preparation.

where $\Lambda$ is a $20 \times 20$ diagonal matrix of subtest standard deviations and $J$ is a $20 \times 10$ matrix of stacked $10 \times 10$ identity matrices $I$:

$$J = \begin{bmatrix} I \\ I \end{bmatrix} .$$

In this model, the diagonal elements of $\Phi$ are constrained to 1.

Maximum Likelihood (ML) estimates of the free parameters specified in (1) can be obtained using the COSAN model implemented by PROC CALIS in SAS. The null hypothesis (that test preparation has no effect on measurement precision) can be tested using a likelihood ratio statistic

$$\mathrm{LR} = -2 \left[ \log L(\hat{\theta}_r) - \log L(\hat{\theta}_u) \right],$$

where $\hat{\theta}_r$ are the ML estimators of free parameters for the restrictive, nested model and $\hat{\theta}_u$ are the estimators for the model without restrictions. Here, the restrictions are imposed by setting $R = I$ (i.e., setting $R$ to an identity matrix) which constrains $\rho^*_{\mathrm{GS}} = \cdots \rho^*_{\mathrm{AO}} = 1$, consistent with the outcome expected if test prep has no effect on measurement errors. The LR statistic has a limiting $\chi^2$ distribution with $df = 10$.